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EFFECT OF COMMON QUADRUPOLES ON THE MAXIMUM BEAM-BEAM LIMITED LUMINOSITY IN LOW-β INSERTIONS

M. Month, BNL

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Asymptotic Luminosity

It has been shown(1) that for p-p collisions with low-β and small crossing angles, the beam-beam limited luminosity saturates with decreasing β approaching an asymptotic value. For round beams, this maximum luminosity can be estimated from

$$L_{max} = \sqrt{2}\,\gamma \left(\frac{I^3 \ \Delta\nu_{max}}{e^3 c r_p \ E\ell} \right)^{1/2} , \tag{1}$$

where I is the average current,

$\Delta\nu$ is the beam-beam tune shift limit,

$E \simeq \gamma\ \varepsilon$ is the normalized emittance (95%),

(2ℓ) is the total space around the collision point containing no magnetic elements,

and γ is the energy in proton mass units.

The asymptotic luminosity can essentially be achieved(1) with a crossing angle, α,

$$\alpha = \left(\frac{2I \ r_p \ell}{\pi e c \gamma \beta^* \ \Delta\nu_{max}} \right)^{1/2} , \tag{2}$$

with β^* given by its optimum value

$$\beta^*_{opt} = \left(\frac{ec \ E\ell \ \Delta\nu_{max}}{4 r_p I} \right)^{1/2} . \tag{3}$$

Note that the meaning of this choice of crossing situation is that choosing β^* lower does not result in increased luminosity.

Effect of Common Quadrupole

If a bending magnet enclosing both beams is inserted at some distance from the collision point, then it will increase the effective crossing angle of the beams. With sufficient strength, the electromagnetic interaction of the beams can thus be contained within an effective length not essentially different from the physical distance to the magnet. The insertion of a quadrupole common to both beams cannot produce such an effect. In fact, under such conditions, the electromagnetic interaction must be enhanced, for one of the beams at best. Another way of saying this is that the beam-beam tune shift is increased. This enhancement of the tune shift depresses the asymptotic luminosity. Because this effect is a direct consequence of the practical fact that the quadrupoles must have non-zero length and assuming a maximum achievable pole tip field for the quadrupoles, then the decrease in optimum luminosity becomes stronger with increasing energy. As we will see, in the case of common quadrupoles, for sufficiently high energy, the asymptotic luminosity,

$$L_{max} \rightarrow \sqrt{\gamma} \quad \text{(common quadrupole)}$$

rather than

$$L_{max} \rightarrow \gamma \quad \text{(Dipole for beam separation)}$$

when a bending magnet is used to separate the beams after collision.

Thin Lens Model

Consider the beam and appropriate dimension such that the first lens is defocusing away from the collision point. This means the β-function quadratic rise in the drift space is magnified by the defocusing quadrupole.

If β^* is the low-β value (at A in Fig. 1), then with the first lens placed at B, a distance ℓ_o away, we have the betatron function values

$$\beta_B = \ell_o^2/\beta^* \ , \quad \alpha_B = \ell_o/\beta^* \ , \quad \gamma_B = \frac{\beta^*}{\ell_o^2} + \frac{1}{\beta^*} \ . \tag{4}$$

The effect of the lens is to increase α_B to

$$\bar{\alpha}_B = t\ell_o/\beta^* \ , \tag{5}$$

where, in order for the implied focusing system of lenses say a doublet, t will be of the order 3 to 6. The integrated quadrupole strength required is

$$q = \frac{(t-1)\ \alpha_B}{\beta_B} = \frac{(t-1)}{\ell_o} \ . \tag{6}$$

Beam-Beam Tune Shift Enhancement

Near the asymptotic limit (low β), the beam-beam tune shift can be approximated by(1) [see Eq. (2)],

$$\Delta\nu = \frac{2I\ r_p\ \ell}{\pi e c \gamma \beta^*\ \alpha^2} \ , \tag{7}$$

valid for a drift ℓ from the collision point. Let us write the tune shift as a sum of two terms, a contribution from the central drift, a distance ℓ_o, denoted Δ_1, and a contribution from the space after the first lens to the next one, a distance ℓ_Q, denoted Δ_2. For the sake of simplicity, we neglect the contributions to the long-range tune shift from the regions between the second lens and the bending magnet which is ultimately required to separate the beams faster than the natural separation due to the crossing angle. We can compensate for this neglect by taking the length ℓ_Q to be greater than the actual quadrupole length, say by some factor on the order of 1.5.

The contribution Δ_1 is simply

$$\Delta_1 = \frac{a\ \ell_o}{\beta^*} , \tag{8}$$

where

$$a = \frac{2I\ r_p}{\pi e c \gamma \alpha^2} . \tag{9}$$

Using Fig. 1, we calculate Δ_2 as the difference of two terms each of which can be computed from the general expression (7):

$$\Delta_2 = a\left(\frac{\ell_Q+\chi}{\beta_L} - \frac{\chi}{\beta_L}\right) = \frac{a\ell_Q}{\beta_L} . \tag{10}$$

The quantity β_L is

$$\beta_L = \beta_B - 2\bar{\alpha}_B\chi + \bar{\gamma}_B\chi^2 , \tag{11}$$

where

$$\bar{\gamma}_B = \frac{1+\bar{\alpha}_B^2}{\beta_B} = \frac{\beta^*}{\ell_o^2} + \frac{t^2}{\beta^*} , \tag{12}$$

and the position of the virtual waist,

$$\chi = \frac{\bar{\alpha}_B}{\bar{\gamma}_B} = \frac{\ell_o}{t(1+h^2)} , \tag{13}$$

with

$$h = \beta^*/t\ell_o . \tag{14}$$

This gives

$$\beta_L = \frac{\beta^*}{t^2(1+h^2)} , \tag{15}$$

which for small h becomes simply

$$\beta_L \approx \frac{\beta^*}{t^2} . \tag{16}$$

The total enhanced tune shift is therefore

$$\Delta_T = \Delta_1 + \Delta_2 = \frac{a \ \ell_o}{\beta^*} \left(1 + \frac{t^2 \ \ell_Q}{\ell_o}\right) . \tag{17}$$

Note that we have assumed that the β function is symmetric around the collision point.

Effect on Luminosity

For ℓ_o sufficiently large, the luminosity is essentially independent of anything in the region of lenses, the contribution to the overlap integral being negligible. This means that the enhancement in the tune shift has the direct consequence of decreasing the beam-beam limited luminosity. To obtain the beam-beam limited luminosity we substitute (17) into (1). If we let L^o_{max} be the asymptotic luminosity neglecting the electromagnetic interaction in the lenses, then the maximum luminosity including it can be written

$$L_{max} = \frac{L^o_{max}}{(1 + t^2 \ \ell_Q/\ell_o)^{1/2}} . \tag{18}$$

Since $\ell_Q \propto \gamma$, and $L^o_{max} \propto \gamma$, we see that for large γ, $L_{max} \propto \sqrt{\gamma}$.

Maximum Luminosity for POPAE at 1 TeV

We use the POPAE parameters given in Refs. (2,3) for 1 TeV. Taking I = 10 A, γ = 1000, E = 10^{-5} rad m, ℓ_o = 12.5 m, and a beam-beam tune shift limit $\Delta\nu_{max} = 5\text{x}10^{-3}$, we obtain, without common lenses, a maximum luminosity [from (1)]

$$L^o_{max} = 2.05\text{x}10^{34} \text{ cm}^{-2} \text{ sec}^{-1} .$$

To achieve this, we require a low-β value computed from (3),

$$\beta^*_{opt} = 0.70 \text{ m},$$

and a total crossing angle from (2),

$$\alpha = 0.85 \text{ mrad}.$$

In the case common lenses are used, the asymptotic luminosity is reduced by the factor given in (18). Calling the gradient G and obtaining the length ℓ_Q from the required quadrupole strength given in (6), then the reduction factor can be written

$$f = \left(1 + \frac{t^2 \ \ell_Q}{\ell_o}\right)^{-\frac{1}{2}} = \left[1 + \frac{(B\rho) \ t^2(t-1)}{G \ \ell_o^2}\right]^{-\frac{1}{2}} . \quad (19)$$

For 1 TeV collisions, $(B\rho) = 33{,}500$ kG·m. Taking G from Ref. (2), G = 875 kG/m, and assuming a value of $t \simeq 5$, we obtain for the luminosity reduction factor, f = 0.20. That is, the maximum attainable luminosity is reduced by about a factor of 5 by the electromagnetic interaction of the beams in the common lenses. If the focusing system can be designed with $t \approx 3$, then f = 0.42, i.e. the luminosity reduction would then be only by the factor 2.33.

Conclusions:

We have shown that the maximum attainable beam-beam limited luminosity is reduced by allowing the beams to interact electromagnetically in the focusing system. In particular the common would be interesting to investigate just what the minimum separation is for a magnet having a field as sketched in Fig. 2 and what the maximum achievable gradient would be.

References

1. E. Keil, CERN Report, ISR-TH/73-48 (1973)

2. W.W. Lee and L.C. Teng, "Insertions for Colliding Beam Storage Rings", 9th International Conference on High-Energy Particle Accelerators, SLAC, May 1974 (To be published)

3. Tentative parameter list prepared by A. G. Ruggiero.

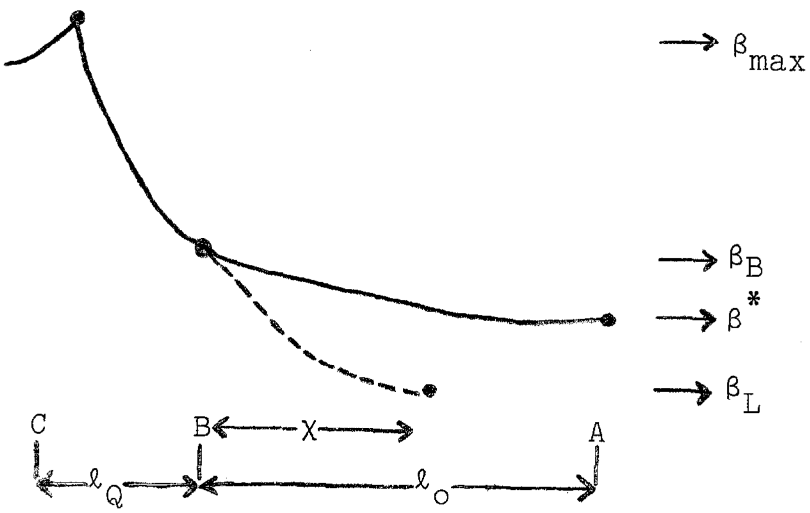

Fig. 1 β-Function Variation Around Collision Region

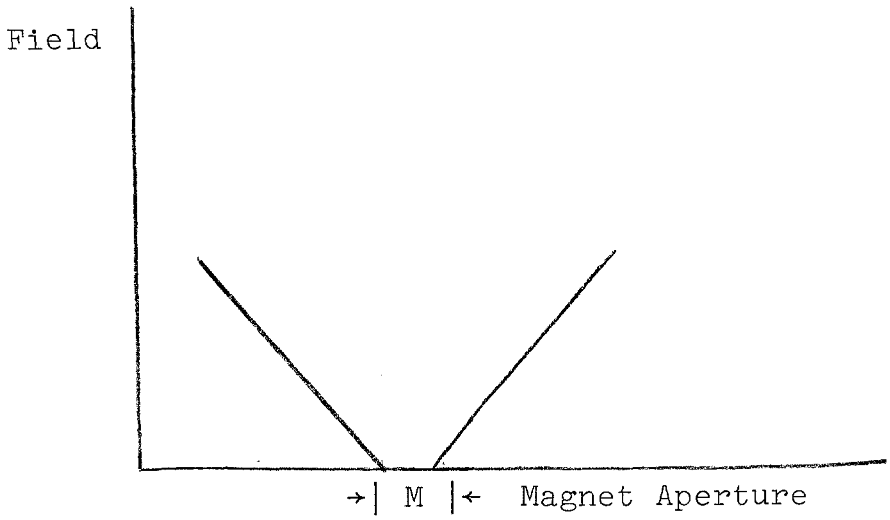

Fig. 2 Bi-quadrupole M is the minimum separation